

CLOSED SUBGROUPS GENERATED BY GENERIC MEASURE AUTOMORPHISMS

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ABSTRACT. We prove that for a generic measure preserving transformation T , the closed group generated by T is a continuous homomorphic image of a closed linear subspace of $L_0(\lambda, \mathbb{R})$, where λ is Lebesgue measure, and that the closed group generated by T contains an increasing sequence of finite dimensional toruses whose union is dense.

1. RESULTS

For a Borel atomless probability measure μ on a standard Borel space, by $\text{Aut}(\mu)$ we denote the group, taken with composition as the group operation, of all measure classes of invertible μ -preserving transformations. The topology on it is the weakest topology making the functions

$$\text{Aut}(\mu) \ni T \rightarrow \mu(TA \triangle A) \in \mathbb{R}$$

continuous for each Borel set A . In this fashion $\text{Aut}(\mu)$ becomes a Polish group. For $T \in \text{Aut}(\mu)$, $\langle T \rangle_c$ will denote the *closed* group generated by T , that is, we set

$$\langle T \rangle_c = \text{closure}(\{T^n : n \in \mathbb{Z}\}).$$

We study the structure of this group for a generic $T \in \text{Aut}(\mu)$, that is, we are interested in properties of this group that are exhibited on a comeager subset of $T \in \text{Aut}(\mu)$. Our results strengthen theorems of Ageev [2] and de la Rue and de Sam Lazaro [12]. They are also related to a question of Glasner and Weiss [6]. Below we say a bit more about it. The methods developed in [2] and in [12] play an important role in our proofs. Additionally, we rely on the results from [1] and [9].

To state our main result, we need to recall some definitions. Let λ be Lebesgue measure on $[0, 1]$. Let $L_0(\lambda, \mathbb{R})$ be the linear space, with pointwise linear operations, of all measure classes of λ -measurable functions from $[0, 1]$ to \mathbb{R} . We equip this space with the topology of convergence in measure. This topology is Polish and makes the linear operations continuous. In some

2010 *Mathematics Subject Classification.* 37A15, 22F10, 03E15.

Key words and phrases. Measure automorphism, F-space, generic objects.

Research supported by NSF grant DMS-1001623.

situations, we consider $L_0(\lambda, \mathbb{R})$, as a Polish group with vector addition as the group operation. Similarly, let $L_0(\lambda, \mathbb{T})$ be the group, with pointwise multiplication, of all measure classes of λ -measurable functions from $[0, 1]$ to the circle group \mathbb{T} . Again, we take it with the topology of convergence in measure. This topology makes $L_0(\lambda, \mathbb{T})$ into a Polish group. There is a canonical continuous homomorphism

$$\exp: L_0(\lambda, \mathbb{R}) \rightarrow L_0(\lambda, \mathbb{T})$$

given by

$$f \rightarrow e^{if}.$$

We will prove the following theorem.

Theorem 1. *Let μ be a Borel atomless probability measure on a standard Borel space. There exists a comeager subset H of $\text{Aut}(\mu)$ such that for each $T \in H$ there exists a closed linear subspace L of $L_0(\lambda, \mathbb{R})$ with the property that $\langle T \rangle_c$ is topologically isomorphic to the subgroup $\exp(L)$ of $L_0(\lambda, \mathbb{T})$.*

In the corollary below we collect a couple of consequences of Theorem 1. Define the topological group

$$\mathbb{T}_\infty$$

by letting it be the direct limit of the sequence \mathbb{T}^n , $n \in \mathbb{N}$, where \mathbb{T}^n is identified with the closed subgroup of \mathbb{T}^{n+1} consisting of elements whose last coordinate is equal to 1. So open sets in \mathbb{T}_∞ are precisely those sets whose intersection with each \mathbb{T}^n is open. Clearly \mathbb{T}_∞ is not a Polish group.

Corollary 2. *There is a comeager set $H \subseteq \text{Aut}(\mu)$ such that for each $T \in H$*

- (i) *$\langle T \rangle_c$ is a continuous homomorphic image of a closed linear subspace of $L_0(\lambda, \mathbb{R})$;*
- (ii) *there is a continuous embedding of \mathbb{T}_∞ into $\langle T \rangle_c$ whose image is dense in $\langle T \rangle_c$.*

In Corollary 2, point (i) strengthens de la Rue and de Sam Lazaro's result from [12] that a generic transformation lies on a one-parameter subgroup of $\text{Aut}(\mu)$. Point (ii) strengthens the result of Ageev [2] that for a generic $T \in \text{Aut}(\mu)$ the group $\langle T \rangle_c$ contains an arbitrary finite abelian group.

Glasner and Weiss in [6] asked if $\langle T \rangle_c$ is a Lévy group for a generic $T \in \text{Aut}(\mu)$. Actually, it is not even ruled out at this point that $\langle T \rangle_c$ is isomorphic to $L_0(\lambda, \mathbb{T})$. Theorem 1 and Corollary 2 provide some evidence that such results may be true. More evidence was subsequently found by Melleray and Tsankov in [10], where they showed that the group $\langle T \rangle_c$ is extremely amenable for a generic $T \in \text{Aut}(\mu)$. One may hope that extreme amenability and an increasing sequence of toruses with dense union, as in Corollary 2(ii), will imply being Lévy, but such an implication is false in

general as shown in [5]. Additionally, it is proved in [10] that the closed subgroup generated by a generic unitary transformation of a separable infinite dimensional Hilbert space is isomorphic to $L_0(\lambda, \mathbb{T})$.

Theorem 1 can be combined with the methods of [2] to obtain also the result of Stepin and Eremenko [13] that $\langle T \rangle_c$ contains the group $\mathbb{T}^{\mathbb{N}}$ for a generic $T \in \text{Aut}(\mu)$. (This theorem, which strengthens an earlier result, announced in [3], that $\langle T \rangle_c$ contains an arbitrary countable abelian group, is stated in the abstract of [13]. It is not proved explicitly in that paper, but Aaron Hill points out that it can be deduced from [13, Theorem 1.3].) Furthermore, using the approach of the present paper and the part of the work of Melleray and Tsankov [10] that pertains to the isometry group of the Urysohn space, one could show that a result analogous to Corollary 2(i) holds for this group (the closed group generated by a generic isometry of the Urysohn space is a continuous homomorphic image of a separable F-space) provided one could show that a generic isometry of the Urysohn space lies on a one-parameter group. However, the question whether this is true remains open.

2. PROOFS

We will need the following lemma that resembles the Kuratowski–Ulam theorem.

Lemma 3. *Let $A \subseteq \text{Aut}(\mu)$ have the Baire property. Then A is meager (comeager, respectively) if and only if there is a comeager set of $T \in \text{Aut}(\mu)$ such that $A \cap \langle T \rangle_c$ is meager (comeager, respectively) in $\langle T \rangle_c$.*

Proof. We will need the following result of King [9, Theorem (23a), p.538]: For $q \in \mathbb{Z} \setminus \{0\}$, the function $f_q : \text{Aut}(\mu) \rightarrow \text{Aut}(\mu)$ given by $f_q(T) = T^q$ has the property that the image of each open non-empty subset of $\text{Aut}(\mu)$ is somewhere dense.

(This theorem is stated in [9] for $q > 1$, but it is clear for $q = 1$, and the case $q < 0$ follows from the case $q > 0$ since $T \rightarrow T^{-1}$ is a homeomorphism of $\text{Aut}(\mu)$.)

The property from the conclusion of this result implies that the images under f_q , $q \in \mathbb{Z} \setminus \{0\}$, of non-meager sets are non-meager. It follows from this statement that preimages under f_q of comeager sets are comeager. Let A be comeager. Let $B \subseteq A$ be a G_δ set dense in $\text{Aut}(\mu)$. Set

$$B_\infty = \bigcap_{q \in \mathbb{Z} \setminus \{0\}} f_q^{-1}(B).$$

Observe that, by what was said above, B_∞ is comeager. We also have that for each $T \in B_\infty$

$$\{T^q : q \in \mathbb{Z} \setminus \{0\}\} \subseteq B.$$

Note that the set of all $T \in \text{Aut}(\mu)$ for which $\{T^q : q \in \mathbb{Z}\}$ is not discrete is a G_δ and it is dense in $\text{Aut}(\mu)$ since it contains all transformations isomorphic to irrational rotations of the circle. Let C consist of those $T \in B_\infty$ for which the set $\{T^q : q \in \mathbb{Z}\}$ is not discrete. Then C is still comeager. It follows, that for each $T \in C$, $B \cap \langle T \rangle_c$ is dense in $\langle T \rangle_c$. Since B is a G_δ , we have that $B \cap \langle T \rangle_c$, and therefore also $A \cap \langle T \rangle_c$, is comeager in $\langle T \rangle_c$, as required.

To complete the proof of the lemma it suffices to show that if A is non-meager, then there exists a non-meager set of $T \in \text{Aut}(\mu)$ such that $A \cap \langle T \rangle_c$ is non-meager. Since A is non-meager, it is comeager in some non-empty open set. This open set contains a transformation with dense conjugacy class by [7, Theorem 1]. Thus, there exist $S_n \in \text{Aut}(\mu)$, $n \in \mathbb{N}$, such that $\bigcup_n S_n A S_n^{-1}$ is comeager. So, there is a comeager set of T such that $\bigcup_n S_n A S_n^{-1}$ is comeager in $\langle T \rangle_c$. It follows that there exists n_0 and a non-meager set of T such that $S_{n_0} A S_{n_0}^{-1}$ is non-meager in $\langle T \rangle_c$. For each such T , A is non-meager in

$$S_{n_0}^{-1} \langle T \rangle_c S_{n_0} = \langle S_{n_0}^{-1} T S_{n_0} \rangle_c.$$

Thus, there exists a non-meager set of T' with A non-meager in $\langle T' \rangle_c$. \square

Let us point out that Lemma 3 can also be derived using the methods of [10, Section 8]; one would use Lemma 8.7, Theorem A.5, and the proof of Theorem 8.13 of that paper. These methods were invented after the authors of [10] became aware of the statements of the above lemma and of Theorem 1.

We will also need the property stated in the lemma below of the map $\exp: L_0(\lambda, \mathbb{R}) \rightarrow L_0(\lambda, \mathbb{T})$ defined in Section 1. This property makes the map into an analogue of the exponential map on Lie groups. The lemma is folklore and follows easily from Mackey's cocycle theorem. We include its proof here for the sake of completeness.

Lemma 4. *If $X: \mathbb{R} \rightarrow L_0(\lambda, \mathbb{T})$ is a one-parameter subgroup of the group $L_0(\lambda, \mathbb{T})$, then there exists a unique element $g \in L_0(\lambda, \mathbb{R})$ such that for each $t \in \mathbb{R}$*

$$X(t) = \exp(tg).$$

Proof. The one-parameter group X , gives rise to a Borel function $\phi: [0, 1] \times \mathbb{R} \rightarrow \mathbb{T}$ such that for each $t \in \mathbb{R}$, the function $\phi(\cdot, t)$ is in the λ -class $X(t) \in L_0(\lambda, \mathbb{T})$ and with the property that for almost all with respect

to the product of Lebesgue measures $(x, r, s) \in [0, 1] \times \mathbb{R}^2$ we have

$$(1) \quad \phi(x, r + s) = \phi(x, r)\phi(x, s).$$

By Mackey's theorem [4, Theorem IV.9] by changing ϕ on a set of measure zero in $[0, 1] \times \mathbb{R}^2$, we can assume that (1) holds for all x, r , and s and that ϕ is still Borel. Now for each $x \in [0, 1]$, $\phi(x, \cdot)$ is a continuous homomorphism from \mathbb{R} to \mathbb{T} . Thus, there exists a unique real number $g(x) \in \mathbb{R}$ such that for all $t \in \mathbb{R}$, $\phi(x, t) = e^{itg(x)}$. It is easy to check that so defined function $g: [0, 1] \rightarrow \mathbb{R}$ is Borel, that it fulfills the conclusion of the lemma, and that it is unique as an element of $L_0(\lambda, \mathbb{R})$. \square

Proof of Theorem 1. We will specify three properties that we will show to hold of a generic $T \in \text{Aut}(\mu)$ and then prove that the conjunction of these properties implies the property of T in the conclusion of the theorem.

Recall that an *F-space* is a topological vector space whose topology is given by a complete metric. For example, $L_0(\lambda, \mathbb{R})$ is a separable F-space.

We fix some notation in the broader context of Polish groups. Let G be a Polish group. Let $\mathcal{L}(G)$ be the space of all one parameter subgroups of G , that is, all continuous homomorphisms from \mathbb{R} to G . There are three elements of the structure of $\mathcal{L}(G)$ we want to point out. All the assertions about them made below are easy to prove and we leave it to the reader to supply these proofs.

1. $\mathcal{L}(G)$ is a Polish space when equipped with the topology it inherits from the space of all continuous functions from \mathbb{R} to G taken with the compact-open topology.

2. The function

$$\mathbb{R} \times \mathcal{L}(G) \ni (r, X) \rightarrow rX \in \mathcal{L}(G)$$

given by

$$(rX)(t) = X(rt)$$

defines a continuous with respect to the topology from point 1 action of \mathbb{R} on $\mathcal{L}(G)$.

3. The function $e: \mathcal{L}(G) \rightarrow G$ given by

$$e(X) = X(1)$$

is continuous.

For $T \in \text{Aut}(\mu)$, let

$$L_T = \{X \in \mathcal{L}(\text{Aut}(\mu)) : \text{image}(X) \subseteq \langle T \rangle_c\}.$$

By points 2 and 3, since $\langle T \rangle_c$ is closed, L_T is a closed subset of $\mathcal{L}(\text{Aut}(\mu))$. By the fact that $\langle T \rangle_c$ is abelian and by point 1, L_T is a separable F-space

with multiplication by reals given by the action of \mathbb{R} on $\mathcal{L}(\text{Aut}(\mu))$ from point 2 and addition given by

$$(X + Y)(t) = X(t)Y(t).$$

We will need the following result of de la Rue and de Sam Lazaro [12], which we state here in our terminology:

the image of $e : \mathcal{L}(\text{Aut}(\mu)) \rightarrow \text{Aut}(\mu)$ is comeager.

Now, it follows from Lemma 3 that there exists a comeager set of T such that $\text{image}(e)$ is comeager in $\langle T \rangle_c$. From this point on, our generic transformation T is assumed to have this property, that is, $\text{image}(e)$ is comeager in $\langle T \rangle_c$.

For an element g of a group G , let $C(g)$ stand for the centralizer of g , that is, $C(g) = \{h \in G : hg = gh\}$. We will need the following result of Akcoglu, Chacon, and Schwartzbauer [1, Theorem 3.3], for another proof of which the reader may consult [10]:

there is a comeager set of $S \in \text{Aut}(\mu)$ such that $\langle S \rangle_c = C(S)$.

(In [1], it is actually proved that $\langle S \rangle_c = C(S)$ holds for all transformations S with strong approximation by partitions. It is not difficult to see, and it follows directly from [8, Theorem 1.1], that the set of all transformations with strong approximation by partitions is comeager in $\text{Aut}(\mu)$.)

Now, it follows from Lemma 3 that there is a comeager set of T such that

$$A_T = \{S \in \langle T \rangle_c : \langle S \rangle_c = C(S)\}$$

is comeager in $\langle T \rangle_c$. From this point on we assume that our T has this property.

We aim to show that $e \upharpoonright L_T$ is onto $\langle T \rangle_c$. If $S \in A_T$ and $X \in \mathcal{L}(\text{Aut}(\mu))$ is such that $e(X) = S$, then

$$\text{image}(X) \subseteq C(S) = \langle S \rangle_c \subseteq \langle T \rangle_c.$$

It follows that $X \in L_T$. Thus, we get

$$e(L_T) \supseteq A_T \cap \text{image}(e).$$

The set on the right-hand side is comeager in $\langle T \rangle_c$, hence $e(L_T)$ is comeager in $\langle T \rangle_c$. Since $e \upharpoonright L_T$ is a group homomorphism, considering L_T as a group with vector addition, we see that $e(L_T)$ is a comeager subgroup of the Polish group $\langle T \rangle_c$. It follows that $e(L_T) = \langle T \rangle_c$.

We view elements of $\text{Aut}(\mu)$ as unitary operators on the Hilbert space $L_2^0(\mu, \mathbb{C})$ of all measure classes of square summable complex functions with zero integral. By [7, Theorem 2], a generic element of $\text{Aut}(\mu)$ does not have non-trivial eigenvectors in $L_2^0(\mu, \mathbb{C})$, and we assume that our T has this property. Now, by the spectral theorem [11, Proposition 4.7.13] applied to the set of operators from $\langle T \rangle_c$, there is a Borel probability measure λ on a standard Borel space and an isometry $U : L_2^0(\mu, \mathbb{C}) \rightarrow L_2(\lambda, \mathbb{C})$ such that,

for each $S \in \langle T \rangle_c$, USU^* is equal to the multiplication by an element of $L_0(\lambda, \mathbb{T})$. We identify USU^* with that element. Note that since T does not have non-trivial eigenvectors, λ is atomless and, therefore, can be assumed to be equal to Lebesgue measure. We see that the function

$$\langle T \rangle_c \ni S \rightarrow USU^* \in L_0(\lambda, \mathbb{T})$$

is a group isomorphism between $\langle T \rangle_c$ and its image and it is also a homeomorphism between $\langle T \rangle_c$ and its image, where $\langle T \rangle_c$ is taken with the topology inherited from $\text{Aut}(\mu)$ while its image is taken with the topology of convergence in λ . Since $\langle T \rangle_c$ is a Polish group, being a closed subgroup of $\text{Aut}(\mu)$, its isomorphic and homeomorphic image is closed. It follows that $\langle T \rangle_c$ can be identified with a closed subgroup of $L_0(\lambda, \mathbb{T})$.

By the above identification, we have a continuous surjective homomorphism

$$(2) \quad e \upharpoonright L_T: L_T \rightarrow \langle T \rangle_c < L_0(\lambda, \mathbb{T}).$$

For every $X \in L_T$, the function mapping $t \in \mathbb{R}$ to $e(X(t))$ is a one-parameter subgroup of $\langle T \rangle_c$ and, therefore, by inclusion (2), of $L_0(\lambda, \mathbb{T})$. By Lemma 4, there exists a unique element $f(X)$ of $L_0(\lambda, \mathbb{R})$ such that $\exp(tf(X)) = e(X(t))$ for all $t \in \mathbb{R}$. It is now easy to check that f gives a Borel, and so continuous, linear function $f: L_T \rightarrow L_0(\lambda, \mathbb{R})$ such that $\exp \circ f = e \upharpoonright L_T$. Now the closure in $L_0(\lambda, \mathbb{R})$ of $f(L_T)$ is a closed linear subspace of $L_0(\lambda, \mathbb{R})$ that is as required by the conclusion of the theorem. \square

To obtain Corollary 2, we need the following lemma. In the proof of the lemma, we will use the following result of Ageev [2, the proof of (1) on p. 217]:

given a natural number $n \geq 1$, the projection on the first coordinate of a relatively open, non-empty subset of $\{(T, S) \in \text{Aut}(\mu)^2: S^n = \text{Id} \text{ and } ST = TS\}$ is non-meager.

Lemma 5. *There is a comeager set of $T \in \text{Aut}(\mu)$ with $\langle T \rangle_c$ containing a dense torsion subgroup.*

Proof. Fix a metric d on $\text{Aut}(\mu)$ compatible with the topology. Fix $\epsilon > 0$. We claim that the set

$$A_\epsilon = \{T \in \text{Aut}(\mu): \exists S \text{ } S \text{ has finite order, } ST = TS, \text{ and } d(S, T) < \epsilon\}$$

is comeager. Let S_0 have finite order, say $n \in \mathbb{N}$. Then the set

$$\{(T, S) \in \text{Aut}(\mu)^2: S^n = \text{Id}, ST = TS, d(T, S_0) < \epsilon, \text{ and } d(T, S) < \epsilon\}$$

is relatively open in the set

$$\{(T, S) \in \text{Aut}(\mu)^2: S^n = \text{Id} \text{ and } ST = TS\}$$

and is non-empty, as it contains (S_0, S_0) . It follows by Ageev's result quoted above that its projection on the first coordinate is non-meager and is included in the ϵ ball around S_0 . So each ball around S_0 contains a non-meager set of T from the set A_ϵ . Since the set of such S_0 (with varying n) is dense, it follows that A_ϵ is comeager. As a consequence, we see that $\bigcap_n A_{1/n}$ is comeager. This intersection consists of transformations T for which there exists a sequence (S_n) of finite order transformations with $S_n T = T S_n$ and $S_n \rightarrow T$ as $n \rightarrow \infty$. Since, by the result of Akcoglu, Chacon, and Schwartzbauer [1, Theorem 3.3] already mentioned in the proof of Theorem 1, for a comeager set of T , the centralizer of T is equal to the closed group generated by T , the conclusion follows. \square

Proof of Corollary 2. (i) This point is immediate from Theorem 1.

(ii) It suffices to show that there are groups $K_n < \langle T \rangle_c$, $n \in \mathbb{N}$, such that each K_n is isomorphic to \mathbb{T} and $\bigcup_n K_n$ is dense in $\langle T \rangle_c$. To see sufficiency of this condition, let L_n be the group generated by $K_1 \cup \dots \cup K_n$. Then L_n is a finite dimensional torus, $L_n < L_{n+1}$ for each n , and $\bigcup_n L_n$ is dense in $\langle T \rangle_c$. Possibly going to a subsequence, we can find linear subspaces $V_1 < V_2 < \dots$ of the subspace of $L_0(\lambda, \mathbb{R})$ that maps homomorphically onto $\langle T \rangle_c$ found in (i), with V_n having dimension n , and continuous homomorphisms $\phi_n: V_n \rightarrow L_n$ with discrete $\ker(\phi_n)$ and with $\phi_{n+1} \upharpoonright V_n = \phi_n$. Inductively, we pick a basis e_1, e_2, \dots of the linear space $\bigcup_n V_n$ so that for each n

$$\ker(\phi_n) = \mathbb{Z}e_1 + \dots + \mathbb{Z}e_n.$$

Now there is an isomorphism $f_n: \mathbb{T}^n \rightarrow L_n$ such that

$$\phi_n \circ g_n = f_n \circ \pi_n,$$

where $\pi_n: \mathbb{R}^n \rightarrow \mathbb{T}^n$ is the exponential map and $g_n: \mathbb{R}^n \rightarrow \mathbb{R}e_1 + \dots + \mathbb{R}e_n$ is the canonical linear isomorphism. Then $f_{n+1} \upharpoonright \mathbb{T}^n = f_n$ if \mathbb{T}^n is considered as a subgroup of \mathbb{T}^{n+1} consisting of points with the last coordinate equal to 1. It follows that the sequence (f_n) induces a continuous embedding from \mathbb{T}_∞ to $\langle T \rangle_c$ that is as required.

It remains to find groups K_n . By Lemma 5, we can pick transformations S_n of finite order such that $S_n \rightarrow T$ as $n \rightarrow \infty$, $S_n T = T S_n$, and $S_n \neq \text{Id}$. Using again [1, Theorem 3.3], we can assume that $S_n \in \langle T \rangle_c$ for all n . By Theorem 1, there are one-parameter subgroups X_n , $n \in \mathbb{N}$, of $\langle T \rangle_c$ with $X_n(1) = S_n$. Let $K_n = \text{image}(X_n)$. \square

Acknowledgement. I would like to thank Aaron Hill for bringing to my attention papers [3] and [13].

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